Title: Clifford algebra in Physics and Complexity

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Abstract:
In 1913 Elie Cartan introduced spinors and, after more than a century, their offsprings continue to blossom. In this research we exploit a formulation of Clifford algebra obtained extending to the whole algebra the Fock basis of the linear space of spinors that allows to express all algebra elements linearly in terms of spinors. In this fashion even vectors (gamma matrices) can be expressed linearly in term of spinors thus supporting the well-known Penrose twistor program that spinor structure is the underlying - more fundamental - structure of Minkowski spacetime.

Within this frame many familiar and well established results can be examined under a different light: for example one can study in detail the complex representations of the Clifford algebra of Minkowski space to discover that not only in the signature R^{3,1} the representation can be reduced to the real case (Majorana spinors) but also in signature R^{1,3} the representation can always be reduced to a quaternionic one and this could show that the two possible signatures are not completely equivalent as is generally thought. Another promising application is the rewriting of bilinear invariants of Dirac equation, that are nothing else that the elements of the algebra, in terms of the new basis made of simple spinors.

The same conceptual frame finds also a surprising application in a different field: in Clifford algebra - like in any associative, unital, algebra - every family of commuting, orthogonal, idempotents generate a Boolean algebra; it is thus possible to write any Boolean expression with n literals in terms of the simple spinors of the Clifford algebra of R^{n,n}. This establishes a precise link between the famous Boolean SATisfiability Problem, the mother of all complex problems, and the algebra of fermions: one can exploit the properties of Clifford algebra to give a purely algebraic formulation of SAT that yields a necessary and sufficient condition for unsatisfiability that, from the computational point of view, is fully equivalent to an algorithm to solve SAT problem. This formulation gives insights in the geometrical and topological properties of the SAT problem that could bring to completely different, non combinatorial, algorithms to tackle SAT and other complex problems.

Recent references:
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