Project: Charting the Space of Conformal Boundary Conditions for a Free Fermion

Conformal field theories (CFTs) play a special role in quantum field theory (QFT), because they capture the high and low energy behavior, and because a generic QFT can be described as a deformation of a CFT. Moreover, they apply to the description of critical phenomena, i.e. second order phase transitions, both in statistical physics and in the quantum regime of zero temperature. Due to the large symmetry group, the dynamics of a CFT is very constrained, leading to the research program of the conformal bootstrap, that aims at computing the observables starting from a set of consistency conditions. This has been applied with remarkable success in recent years to several interesting CFTs [1]. More ambitiously, one could hope to find all the possible solutions to these consistency conditions, therefore classifying all possible CFTs. In practice this is still too hard to do with the present techniques and without some further simplifying assumption.

A special subset of CFTs that is simple enough to attack, yet seemingly rich enough, is that of the conformal boundary conditions for free fields. When we have a field propagating on a space with a boundary at weak coupling, we typically impose either Dirichlet or Neumann conditions. However more general boundary conditions encompass interactions of the boundary mode of the field with degrees of freedom localized on the boundary, and such interactions might drive the boundary to criticality, thus defining an *interacting conformal boundary condition*. In applications to statistical physics or condensed matter physics, conformal boundary conditions can be understood as describing surface transitions that happen at the boundary of the material. The fact that the bulk field is free entails additional strong constraints on the possible interactions of its boundary modes, besides the ones due to conformal symmetry. A study of these constraints in the case of a free Maxwell field -for which a rich set of interacting boundary conditions is known to exist- was initiated in [2]. For a free scalar field, an interesting set of interacting boundary conditions was studied recently in [3] and a systematic study of the constraints, together with a numerical study of the possible solutions, is done in [4].

The aim of this project is to apply these techniques to the case of a free fermion in the bulk. The project has a first part which is analytical, and a possible extension to a second part which requires numerics. Like in the case of the scalar and the vector field, the first part requires to first understand the bulk-to-boundary operator product expansion of the free fermion, and then study the constraint imposed by the bulk OPE, which is simply controlled by the free theory, on the boundary correlation functions. The goal is to derive analytical constraints on the boundary three-point functions. Once this is understood, the problem of finding interacting boundary conditions can be attacked with the technique of the numerical bootstrap.

References

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